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On the generation of stochastic simulations of rainfall in space and time for hydrological applications

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International Commission on Statistical Hydrology
International Association of Hydrological Sciences

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Talk outline

- Statistical properties of rainfall
- Elements of a stochastic simulation framework
- STEPS as an example
- Conclusions

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Statistical properties of rainfall

- Moments are a function of scale - $\langle r_l^q \rangle \sim (L/l)^{K(q)}$ where L is the size of the observation domain, and r_l^q is the rain field averaged over scale l and then raised to the power *q*.
- Probability distribution is a function of scale - $Pr(r > \lambda^{\gamma}) \propto \lambda^{-c(\gamma)}$ where $\lambda = {}^{L}/_{l}$
- Power spectrum has a power law - $E(k) \propto k^{-\beta}$
- Temporal evolution is a power law function of scale
- Spatial anisotropy is a function of scale
- Statistical properties of accumulations depend on Lagrangian evolution of the field and the advection, which also varies in space and time
- All of the above varies in space and time

Isotropic power spectral density of rainfall

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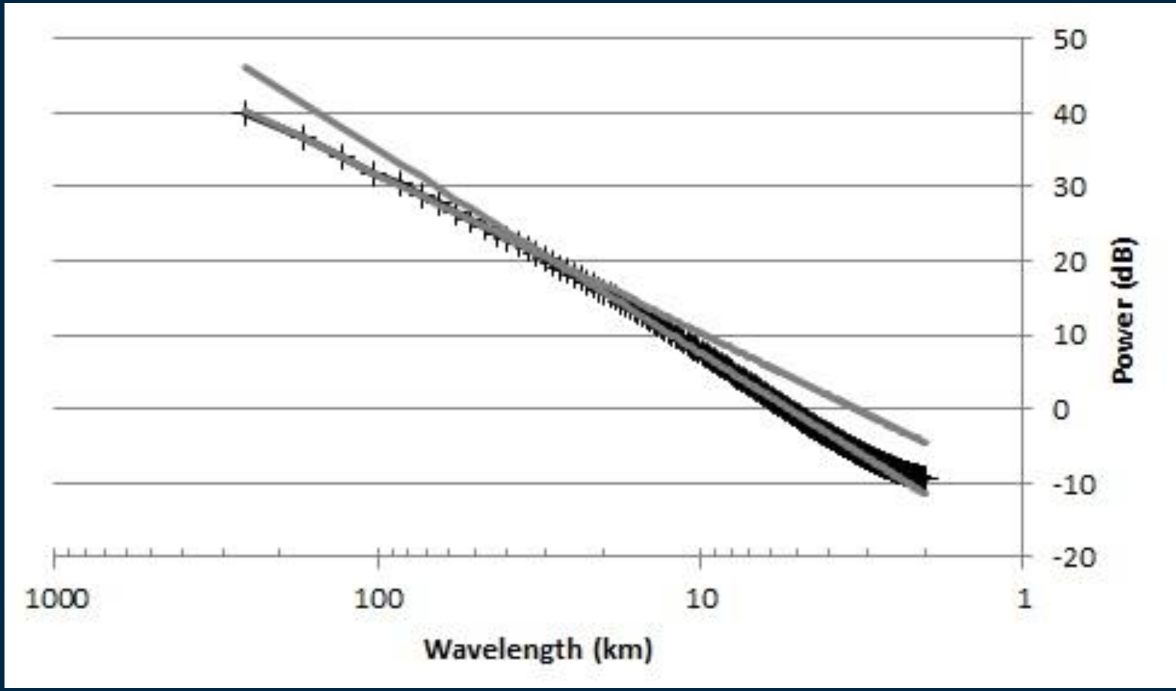
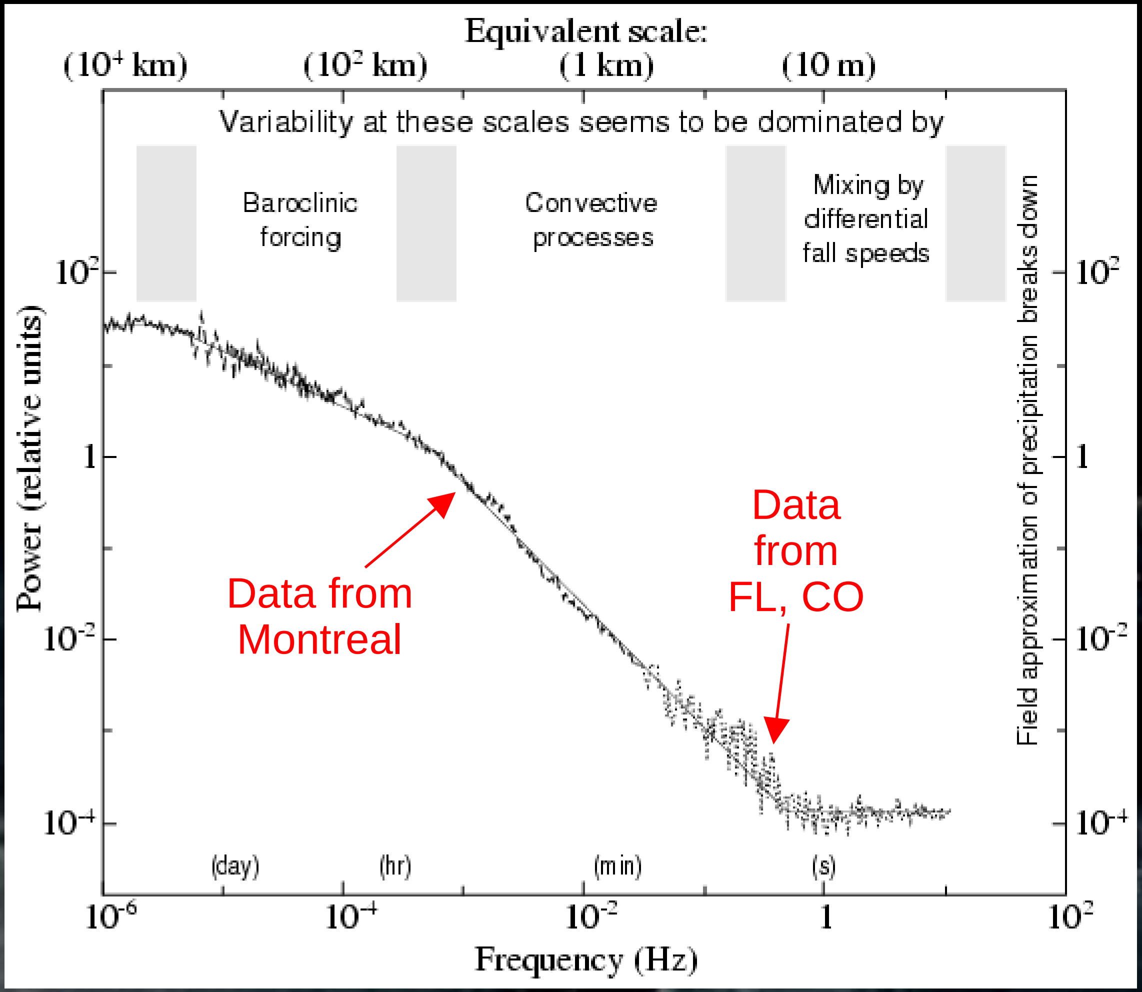
$$E(k) \propto k^{-\beta}$$



Mean isotropic power spectrum of 1 year of 6 min, 1 km radar data in Sydney
Note the change of slope in power spectrum at around 30 km

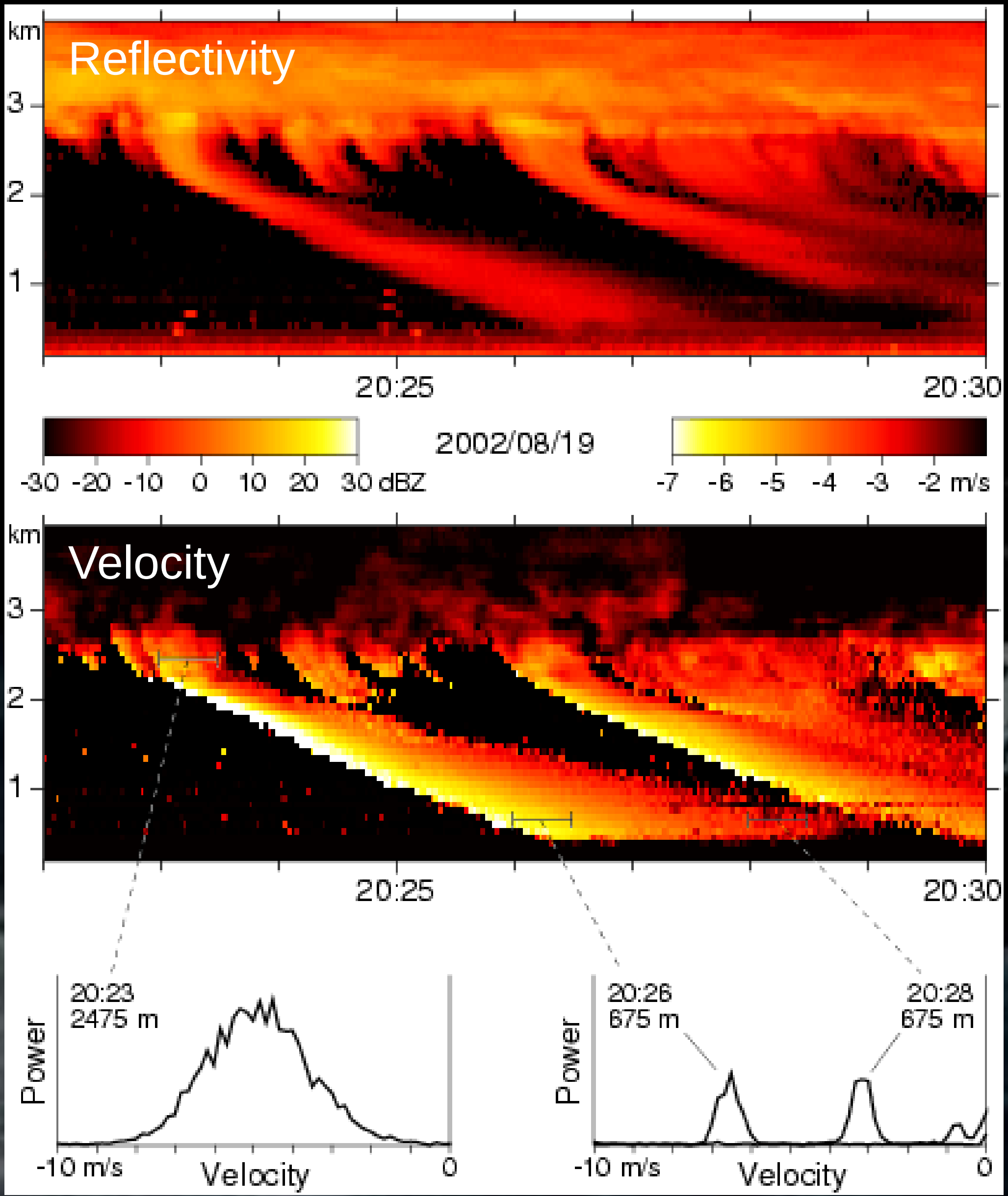
Structure of Rainfall at Small Scales

Fabry, McGill University, 2004



- Below a critical scale (10-200 m (?), event-dependent), precipitation may be considered random [my opinion, others disagree].
- Above that scale, precipitation structure appears to have an organization similar to that of the energy cascade of wind.

(Probable) Physical Origin of the Structure of Rainfall at Small Scales



Fabry, McGill University, 2004

Raindrops are not passive tracers:

- They have inertia;
- They have a size-dependent fall speed.

These two effects cause a randomization of the position of raindrops at small scales that tends to destroy the structure built by wind.

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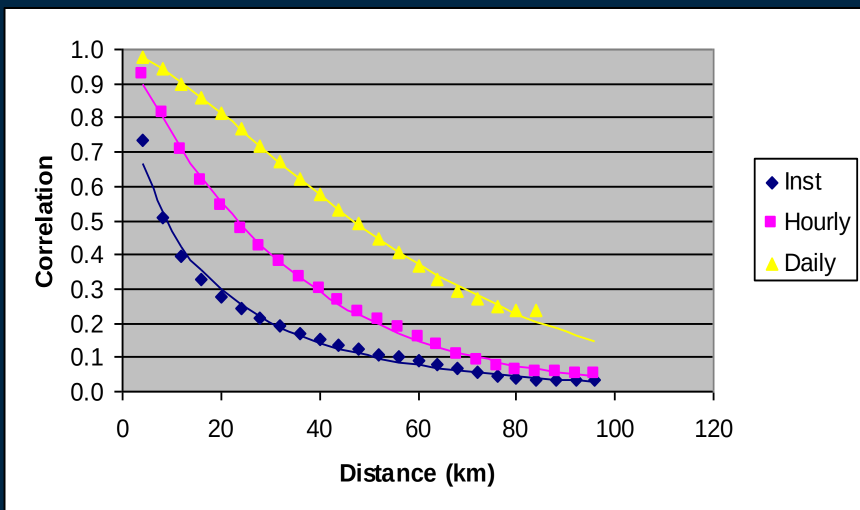
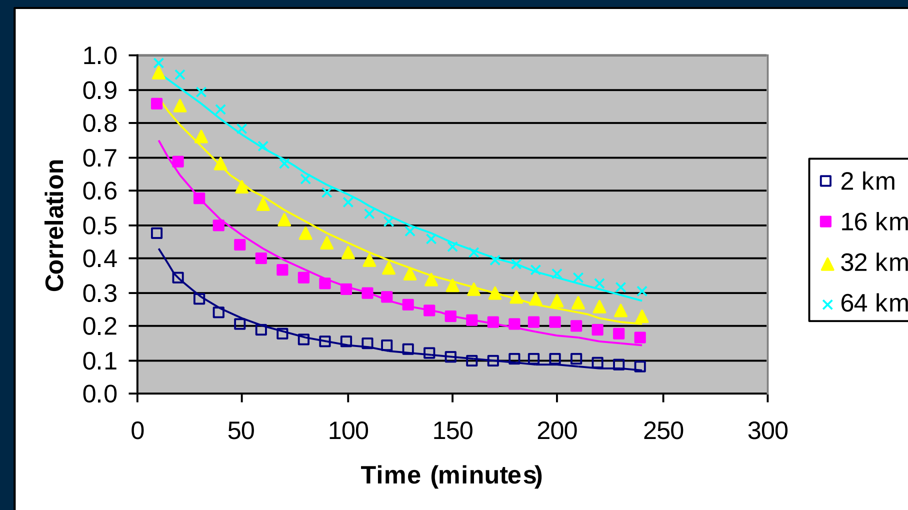
Correlation Function

$$r = e^{-}\left[[\alpha d]^{\beta}\right]$$





2 km	$1/\alpha$	β
10 min	15.1	0.68
Hourly	32.8	1.06
Daily	60.7	1.41

10 min	$1/\alpha$	β
2 km	16	0.36
16 km	79	0.60
32 km	132	0.77

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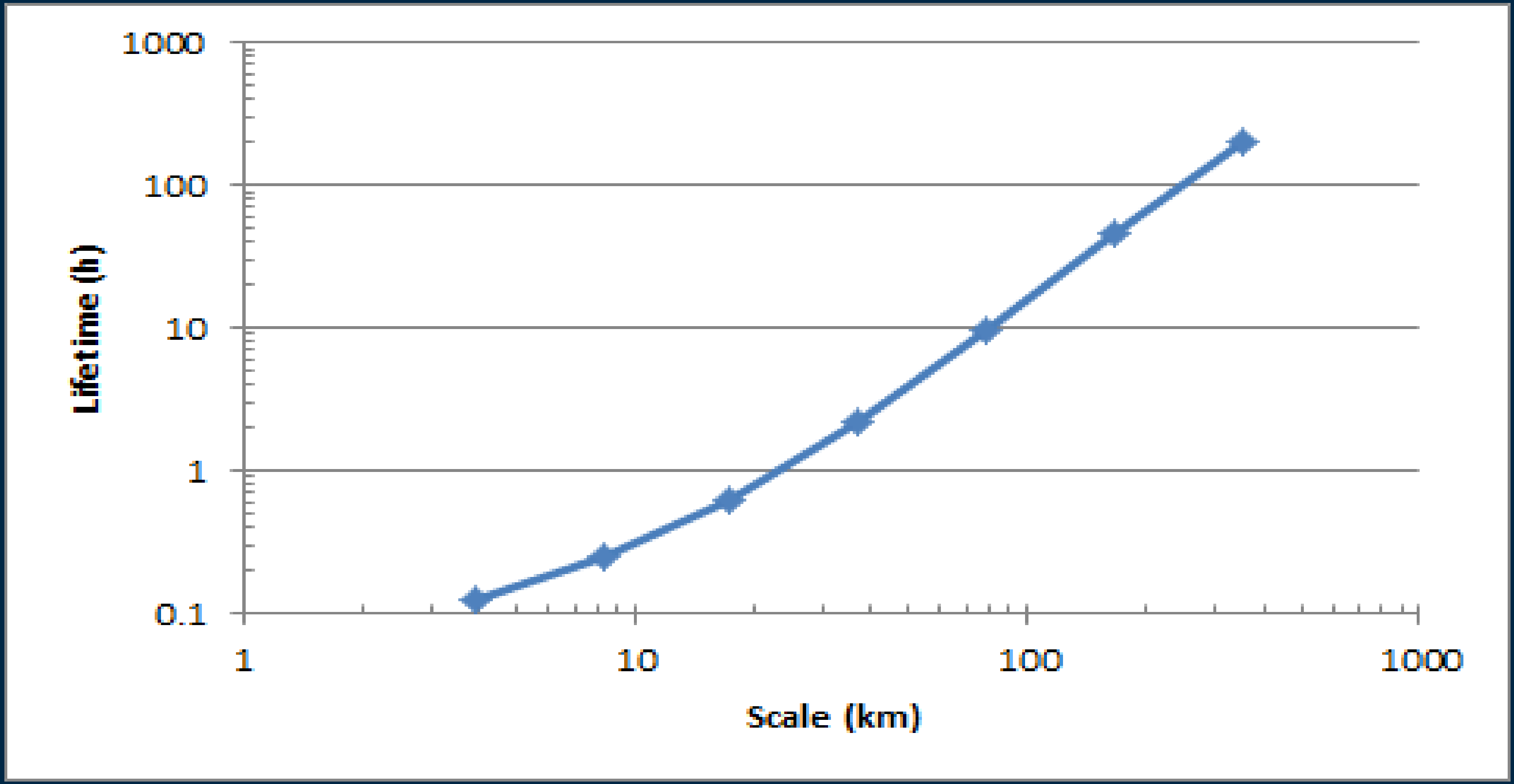
Space –time anisotropy

$$\tau_l \propto l^{(1-H_t)}$$

H_t = space-time anisotropy exponent

If you double the space scale you increase the time scale by ~ 1.6



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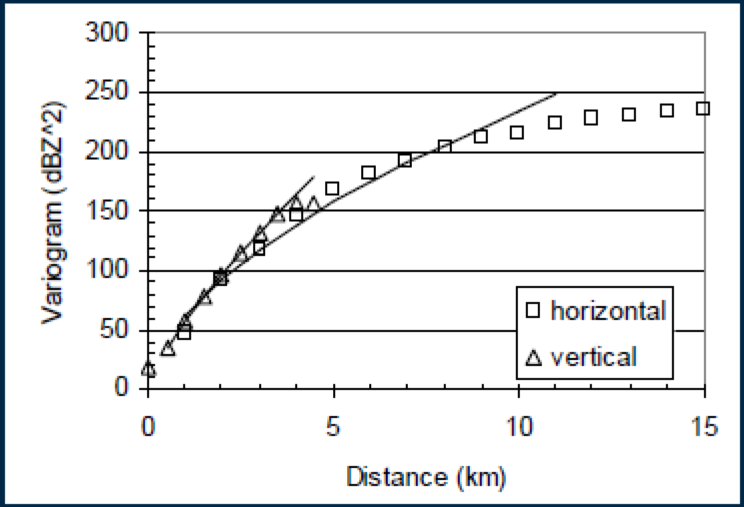
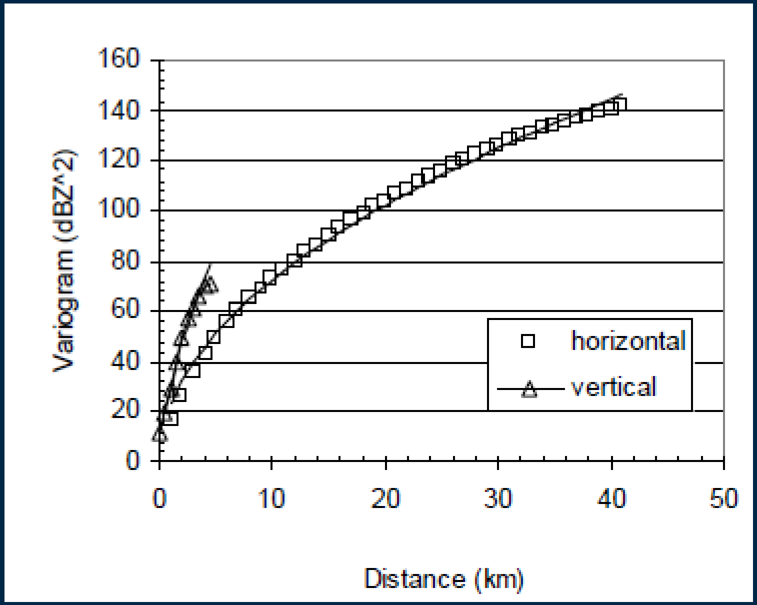
Vertical – horizontal anisotropy



Convective rain



Widespread rain

Seed and Pegram: HAWR, 2001

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Spatial anisotropy

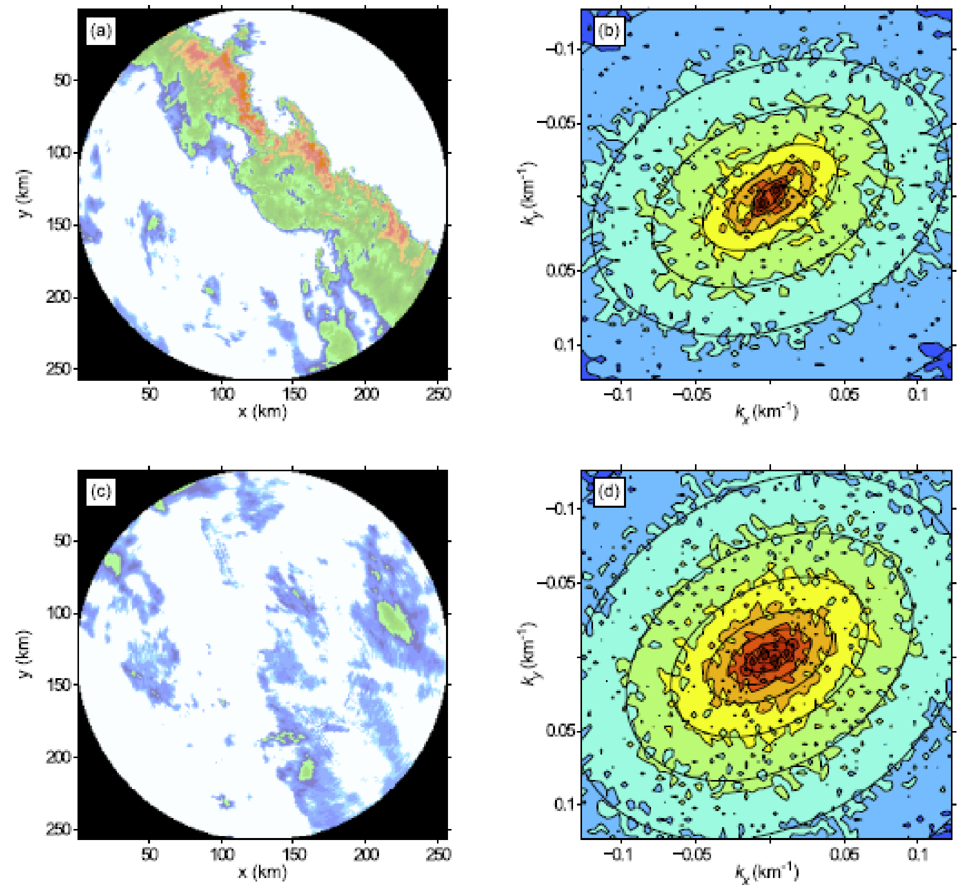

Figure 13. Measured reflectivity field windowed with a boxcar window (a) at 07:00 UTC and (c) at 14:30 UTC. (b) Contours of P_{avg} near the largest scales with selected B_λ (ellipses) corresponding to estimated $\mathbf{G}=\mathbf{G}(-0.132, -0.364, 0.038)$ and $l_\lambda=2.48$ km at 07:00 UTC, and (d) $\mathbf{G}=\mathbf{G}(-0.004, 0.228, 0.101)$ and $l_\lambda=2.04$ km at 14:30 UTC.

Niemi et al WRR 2014

Power Spectra as a function of meteorology

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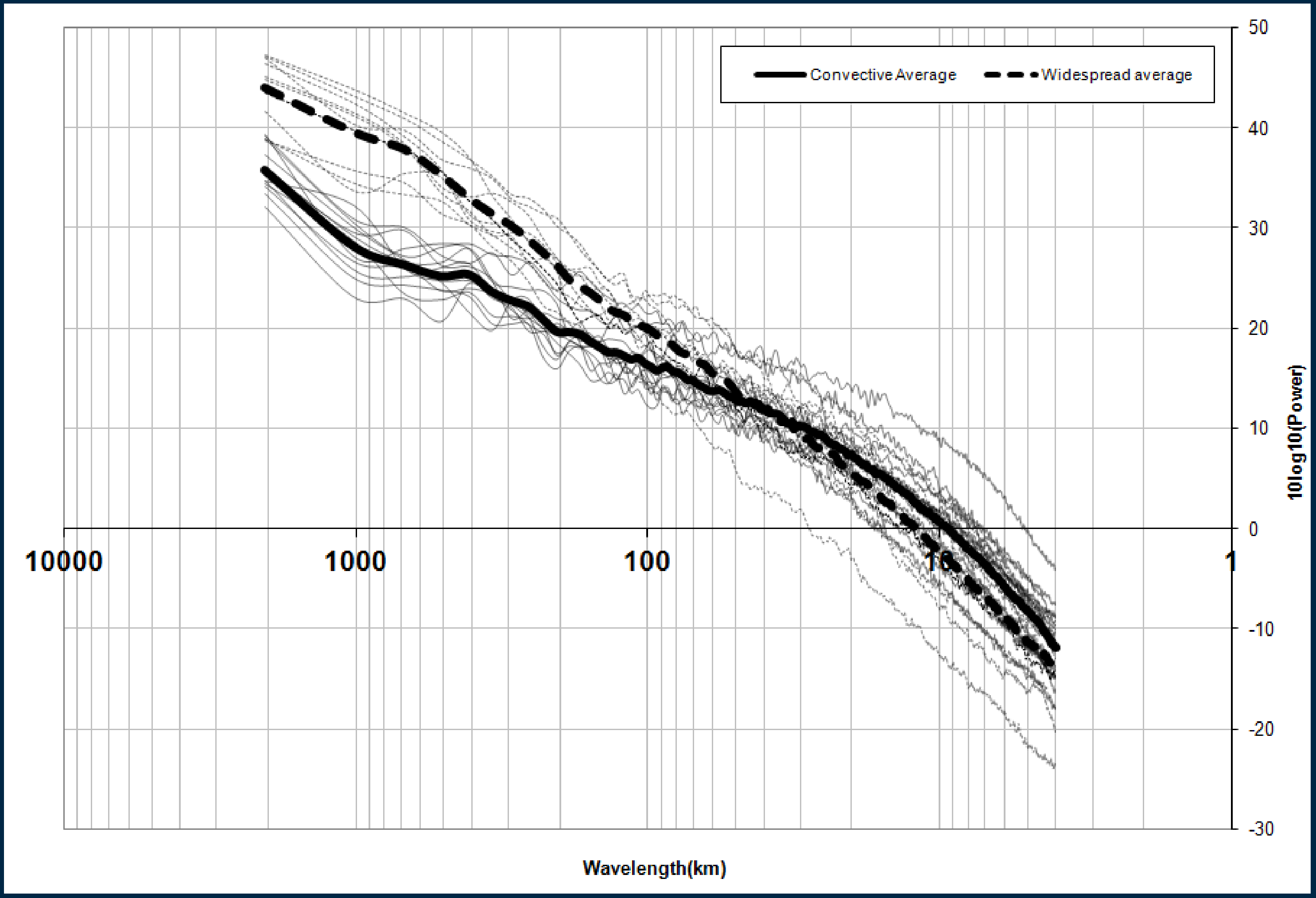

Seed, Pierce, Norman: Water Resources Research 2013

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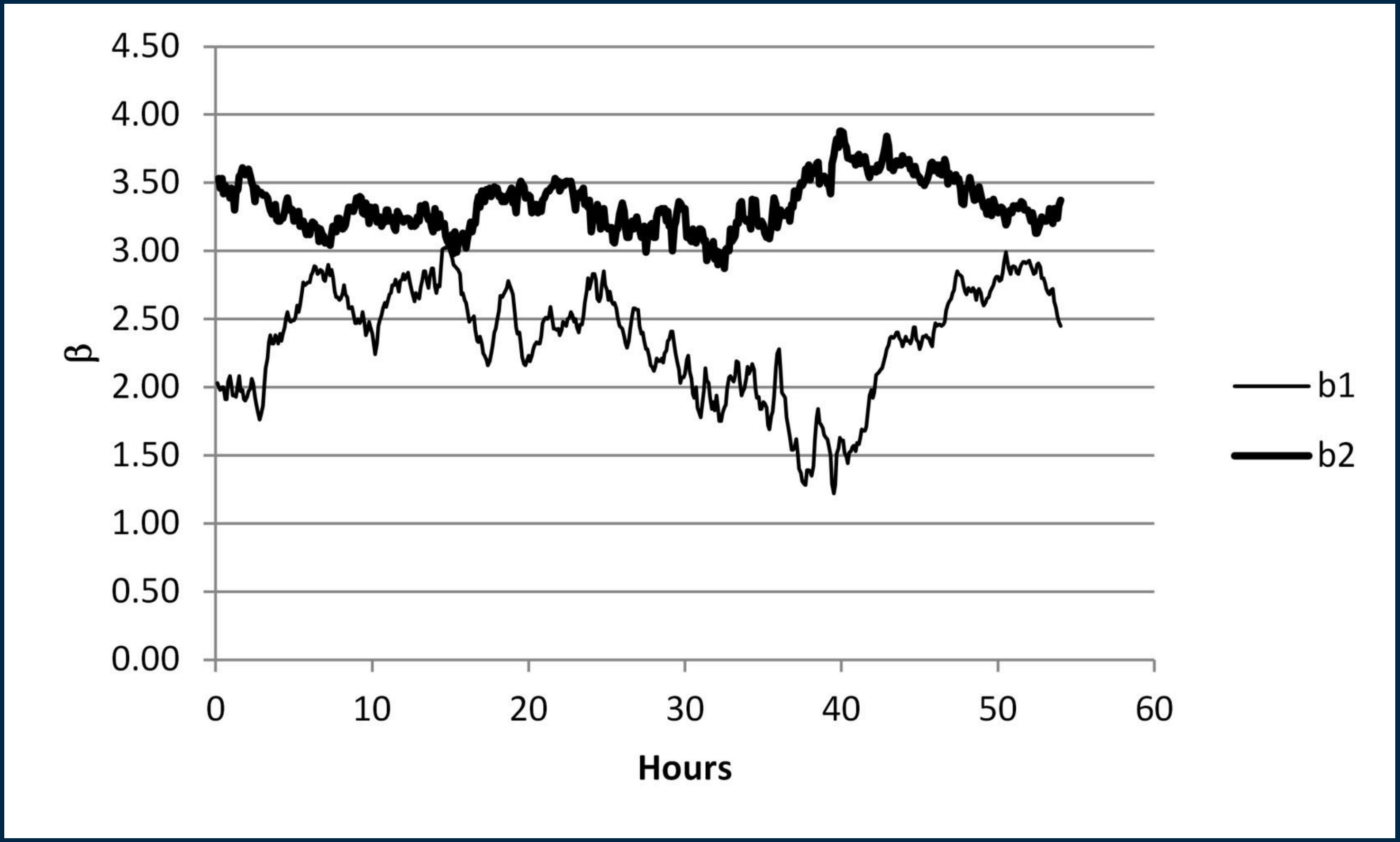
Non-stationarity in time

Time series of β over a 256 km radar image during an event



Probability distribution as a function of location

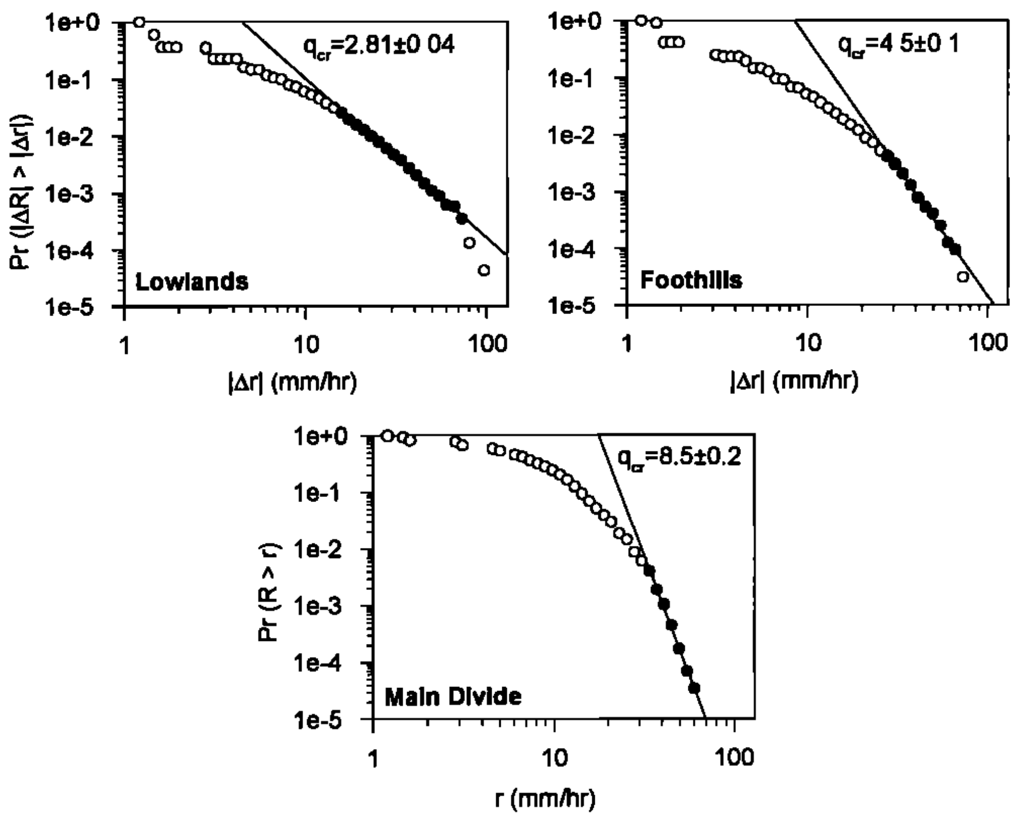


Figure 8. Hyperbolic tail estimates. To remain consistent, the first two depict tail estimates for fluctuations, while near the main divide rain rates were used. The main divide tail is extremely steep indicating much calmer behavior than usually expected for rain in general.

Harris et al, JGR 1996

Non-stationarity in space

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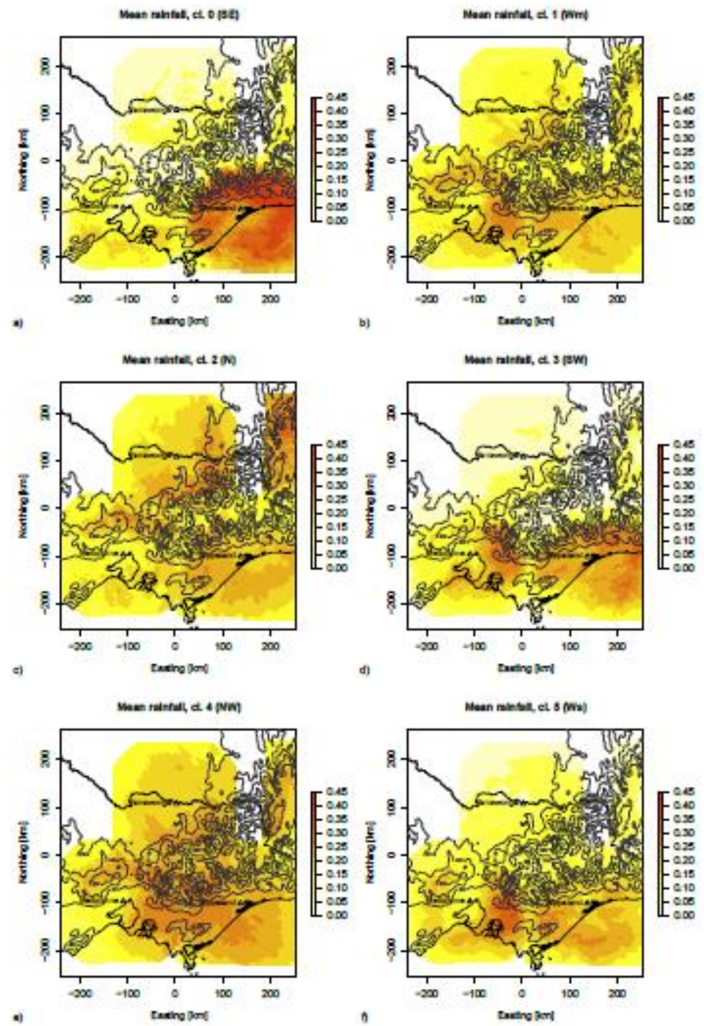

Figure 6. Conditional mean 10 min rainfall accumulations for flow regimes (a) SE, (b) Wm, (c) N, (d) SW, (e) NW and (f) Ws.

Dynamic scaling

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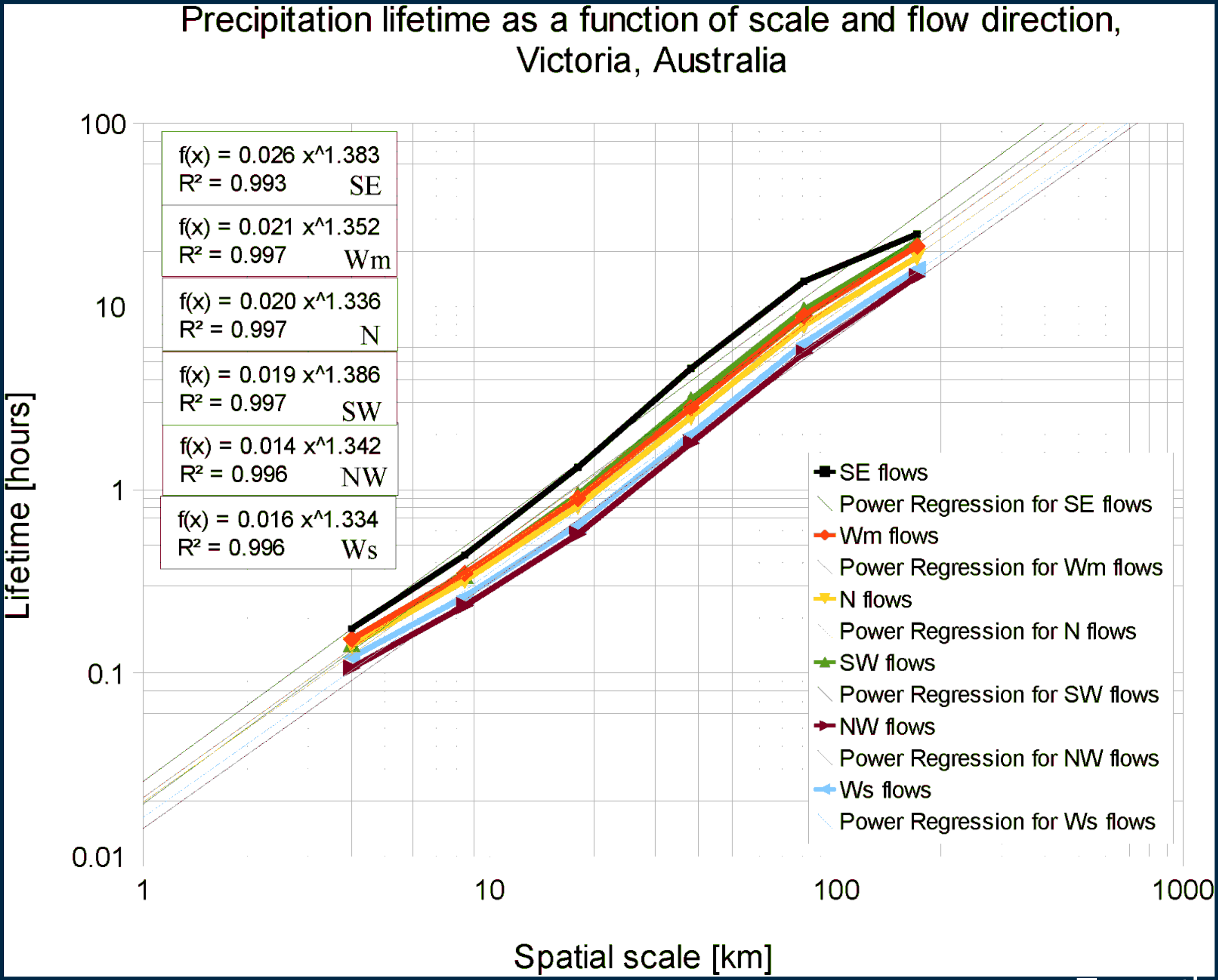

Foresti et al, 2014

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Elements of a stochastic space-time rainfall model

- Rainfall generator of random fields with at least a log normal distribution and scaling structure
- Temporal updater of the spatial field in Lagrangian coordinates, scale dependent
- Advection generator and updater – needs to be a field if working on a domain > 100 km
- Models to generate time series of system parameters (both within and between storms)

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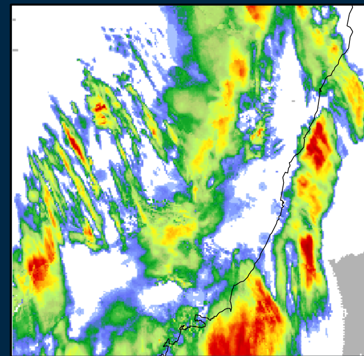
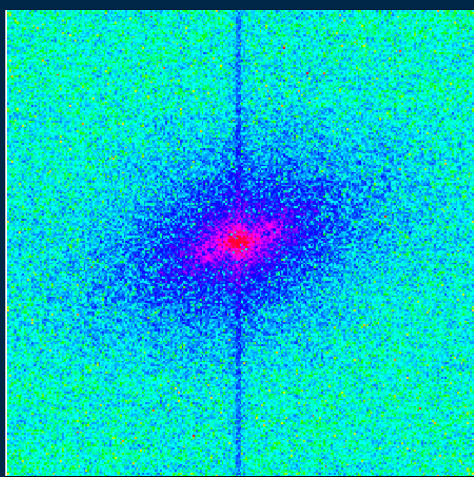
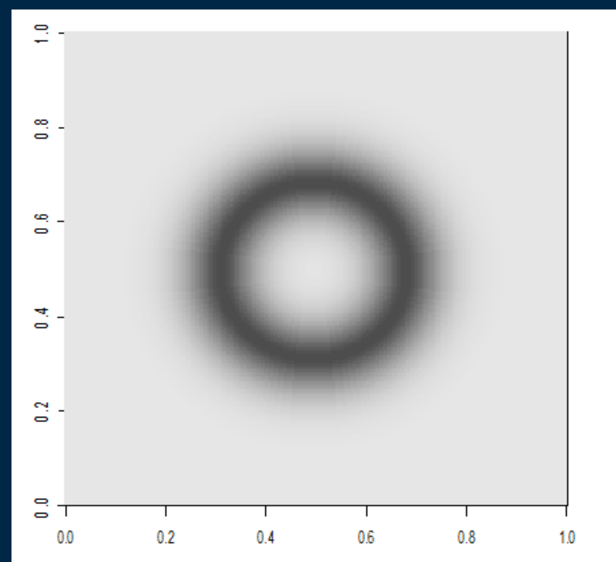
Basis function



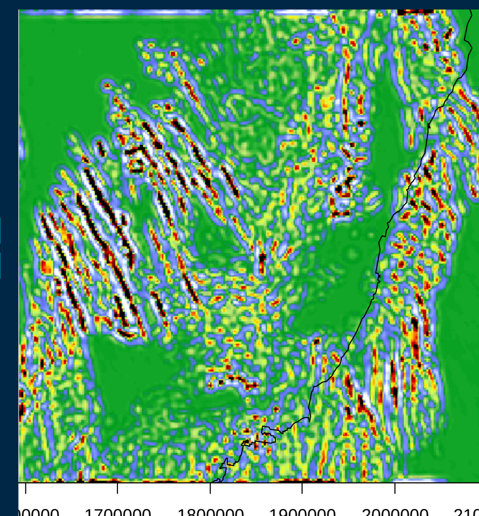
Rainfields3 rainfall for 14:00 hrs UTC 02 Sep 2016



2D Power Spectrum

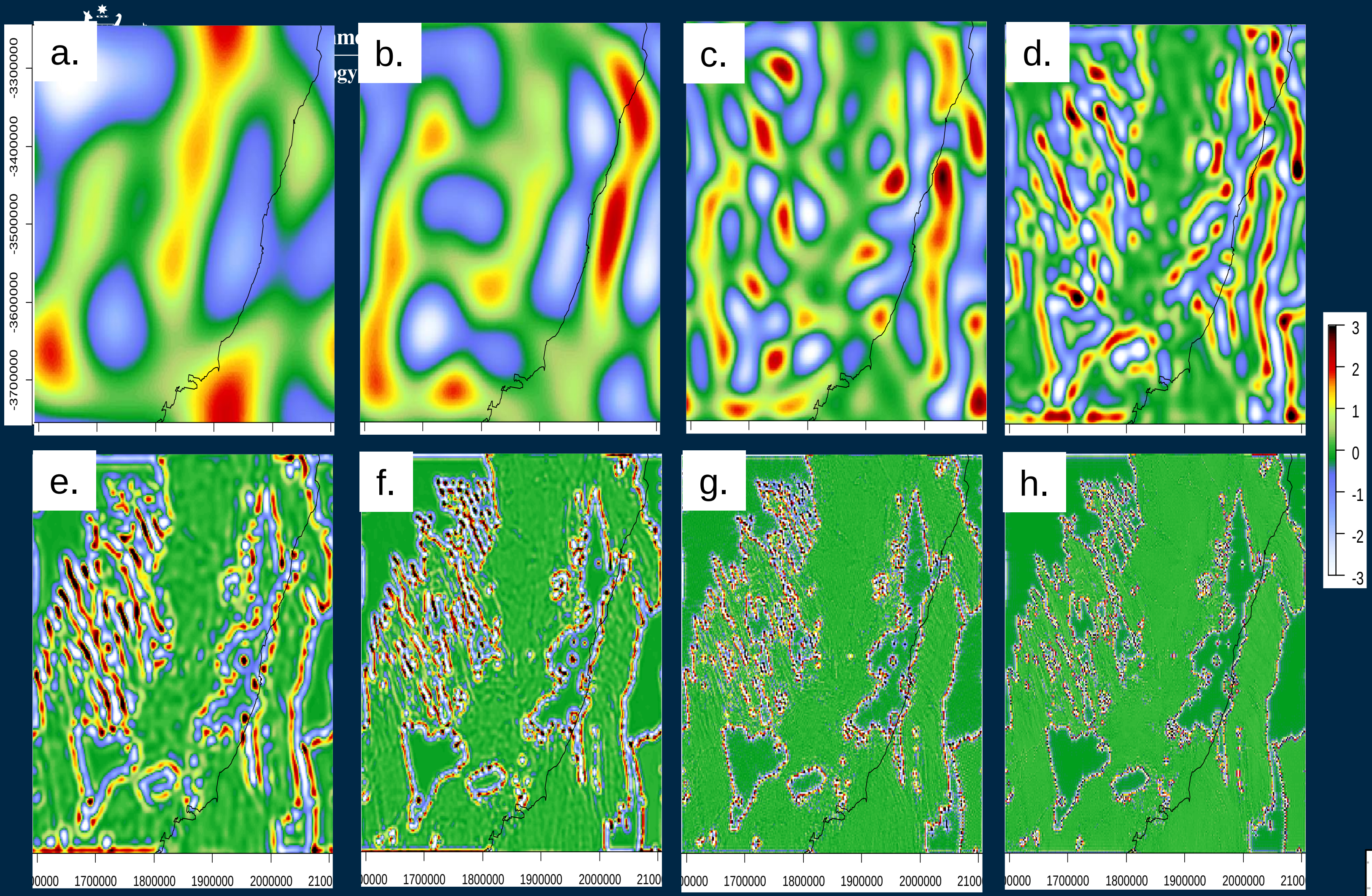


Filter
16-32 km



Filtered image
after inverse FFT

Renzullo et al 2017



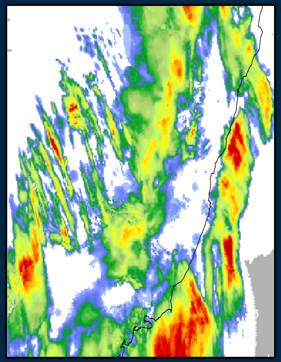


Fig. Cascade levels derived from Rainfields rainfall for 14:00 hrs UTC 02 Sep 2016 normalised to mean = 0 and variance = 1: spatial scales of (a) 256 – 512 km; (b) 128 – 256 km; (c) 64 – 128 km; (d) 32 – 64 km; (e) 16 – 32 km; (f) 8 – 16 km; (g) 4 – 8 km; and (h) 2 – 4 km.

Renzullo et al 2017

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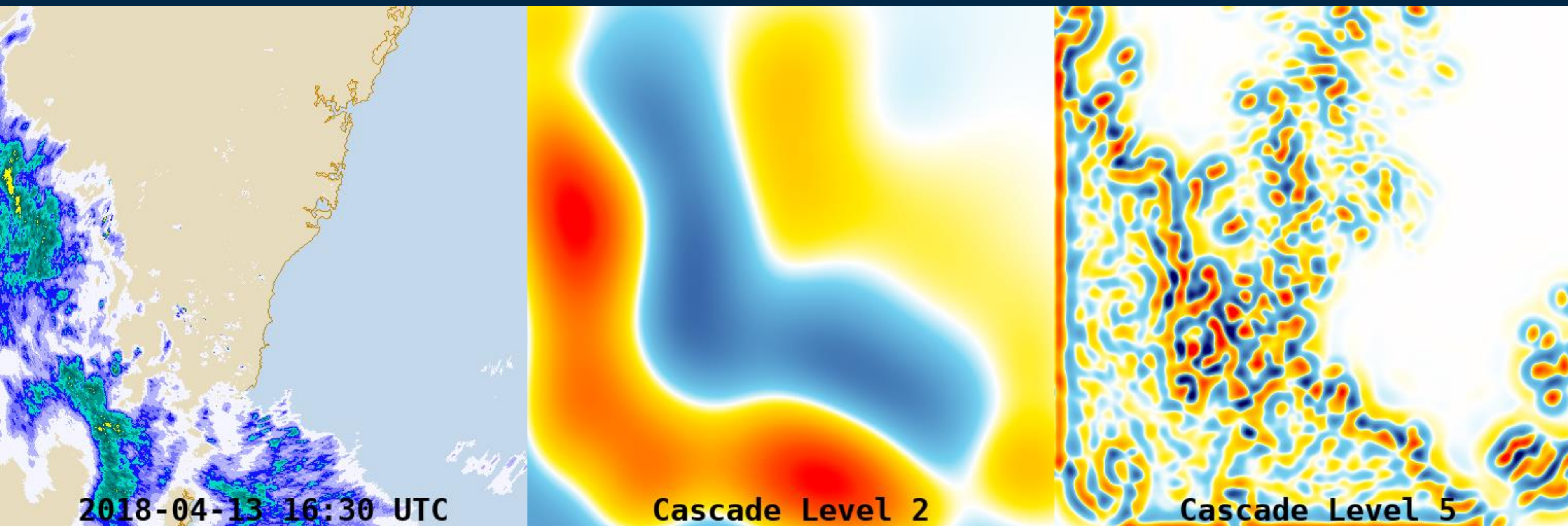
Cascade evolution



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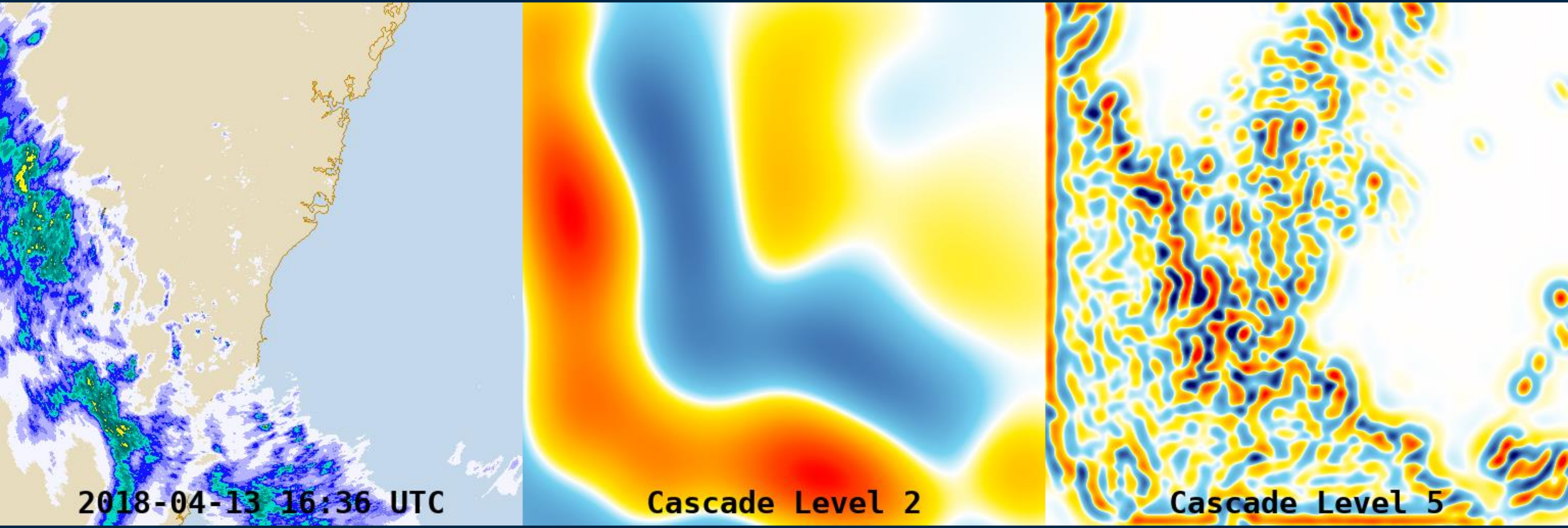
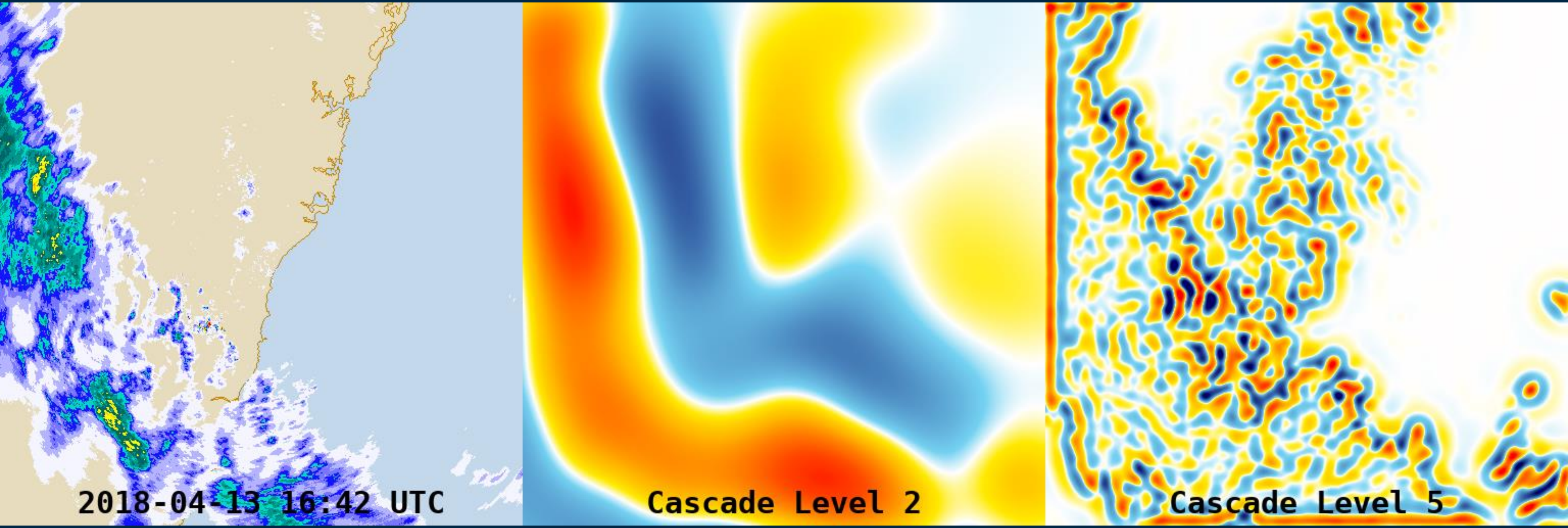
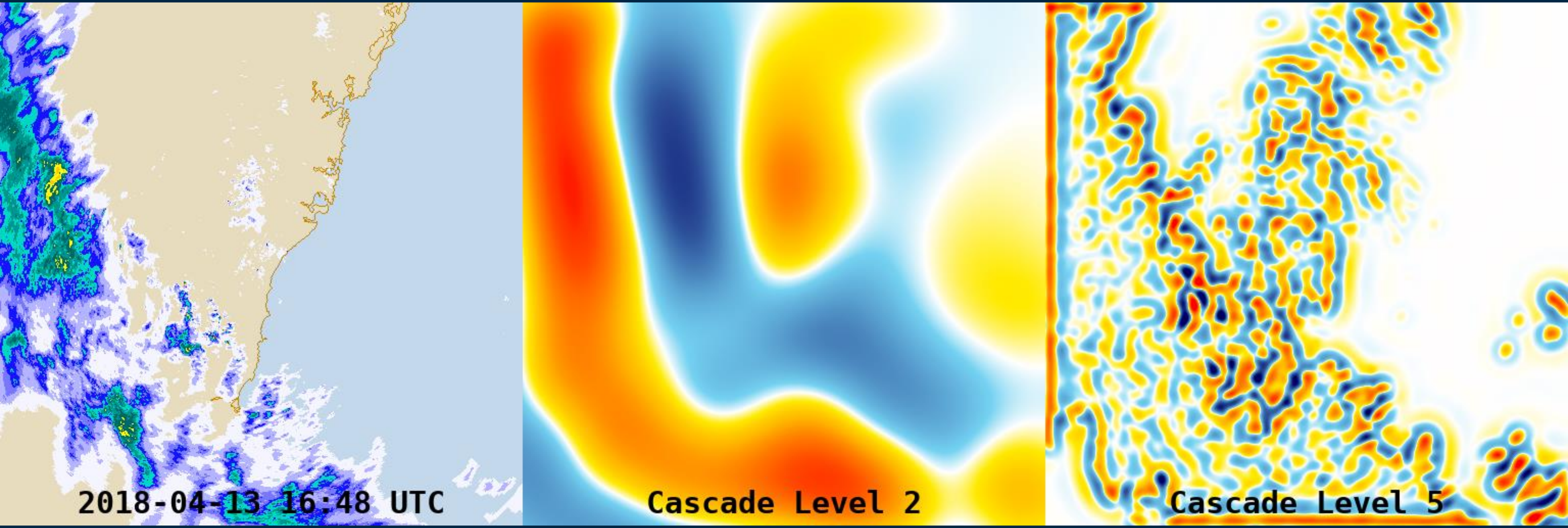
Cascade evolution



Cascade evolution



Cascade evolution



Cascade evolution

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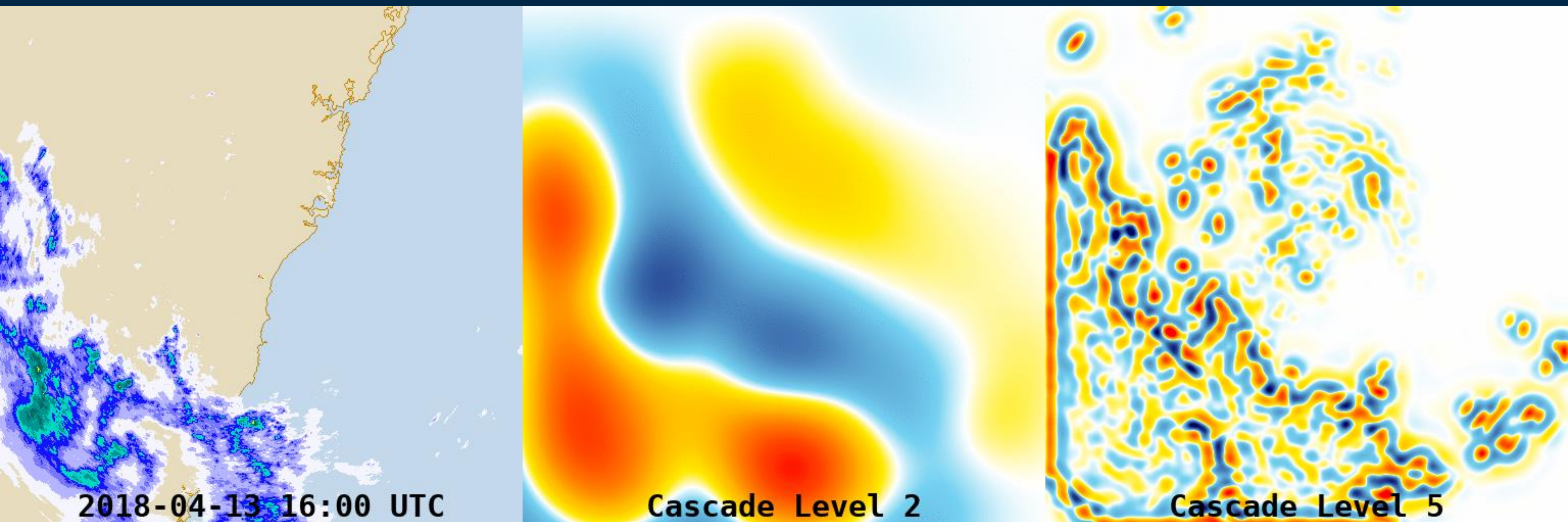

Multiplicative cascade: space

$$\mathbf{z}[x,y] = \mu + \sum_{k=0}^{N} \boldsymbol{\sigma}[k]\mathbf{w}[k,x,y]$$

$\mathbf{z}$ is the field of radar reflectivity (dBZ)

$\mathbf{w}[k]$ is the field of N(0,1) with wavelength $l = Lq^{k}$

$L =$ domain size (km)

$q =$ ratio of scales between cascade levels $k+1, k$

$$\boldsymbol{\sigma}[k] = \boldsymbol{\sigma}[0]q^{kh}, q < 1$$

$\boldsymbol{\sigma}[k]$ is the standard deviation of level k

Multiplicative cascade: space & time

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Temporal evolution

$$\mathbf{w}[t,k,x,y] = \phi_1[k]\mathbf{w}^1[t-1,k,x,y] + \phi_2[k]\mathbf{w}^2[t-2,k,x,y] + \phi_0[k]\boldsymbol{\varepsilon}[k,x,y]$$

where

$\mathbf{w}^n = \mathbf{w}$ that has been advected forwards by n steps through the advection $\mathbf{u}, \mathbf{v}$

$\boldsymbol{\varepsilon}$ is $N(0,1)$ noise with power law filter

$$f_{x,y} = \left[\frac{\omega_{x,y}}{\omega_0}\right]^{\left[\frac{\beta}{2}\right]}$$ and

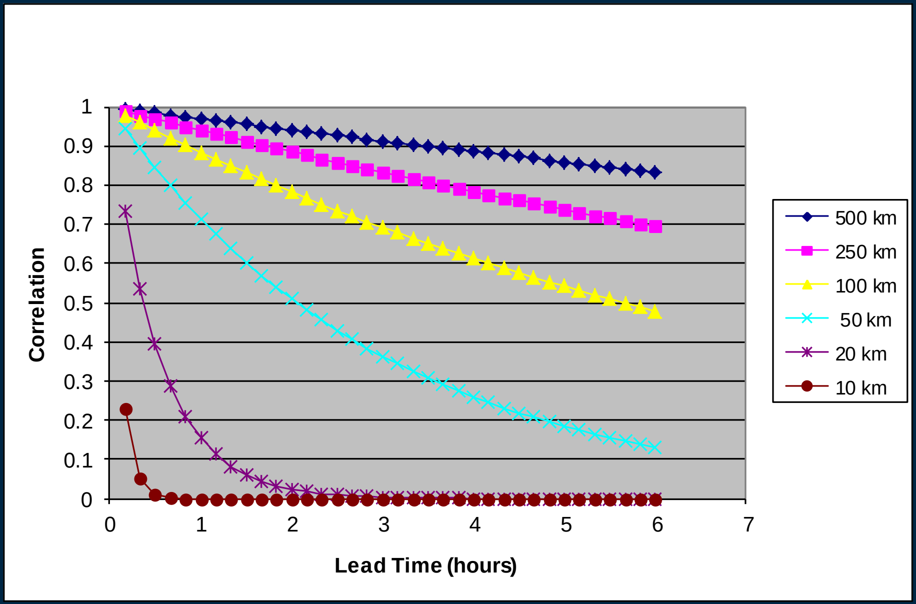
then passed through a notch filter at wave length $\lambda_k = Lq^k$

Estimating the auto-correlation parameters

- For each level in the cascade
  - Advect the level from the previous time forwards
  - Calculate the correlation ρ between t-1 and t for each level
  - Use Yule-Walker equations to calculate ϕ for each level

$$\phi_1[k] = \frac{\rho_{k1} - \rho_{k1}\rho_{k2}}{1 - \rho_{k1}^{\;2}}$$

$$\phi_2[k] = \frac{\rho_{k2} - \rho_{k1}^{\;2}}{1 - \rho_{k1}^{\;2}}$$



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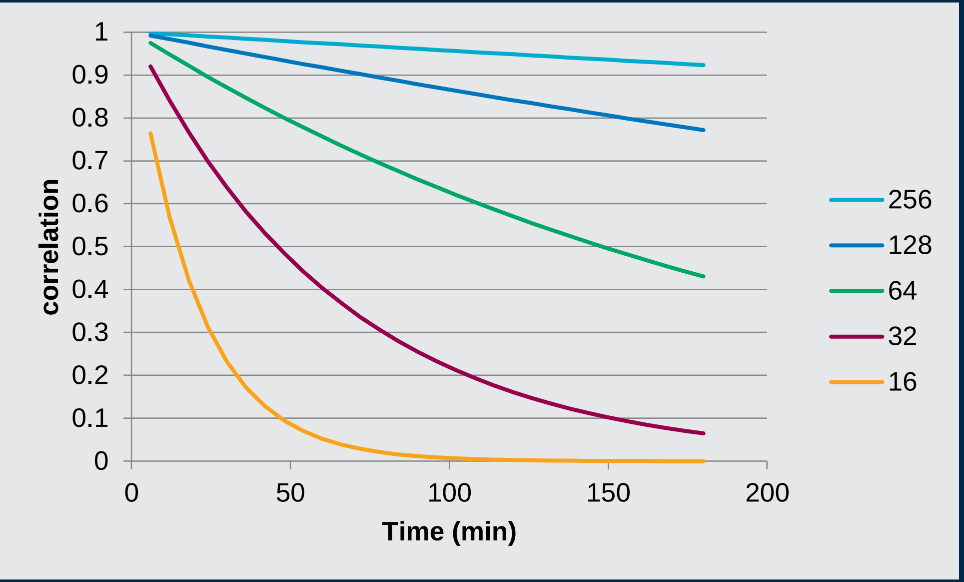
Model for auto-correlation parameters

$$T_l = a l^b \text{ where}$$

T_l is the life time at scale l

$$\rho_l(1) = \exp\left(-\frac{\Delta t}{T_l}\right)$$

$$\rho_l(2) = \rho_l(1)^c$$



Typical values are
a = 0.2
b = 1.7
c = 2.1

Model for spatial scaling

$$\sigma_k = \sigma_0 q^{kH_s}$$

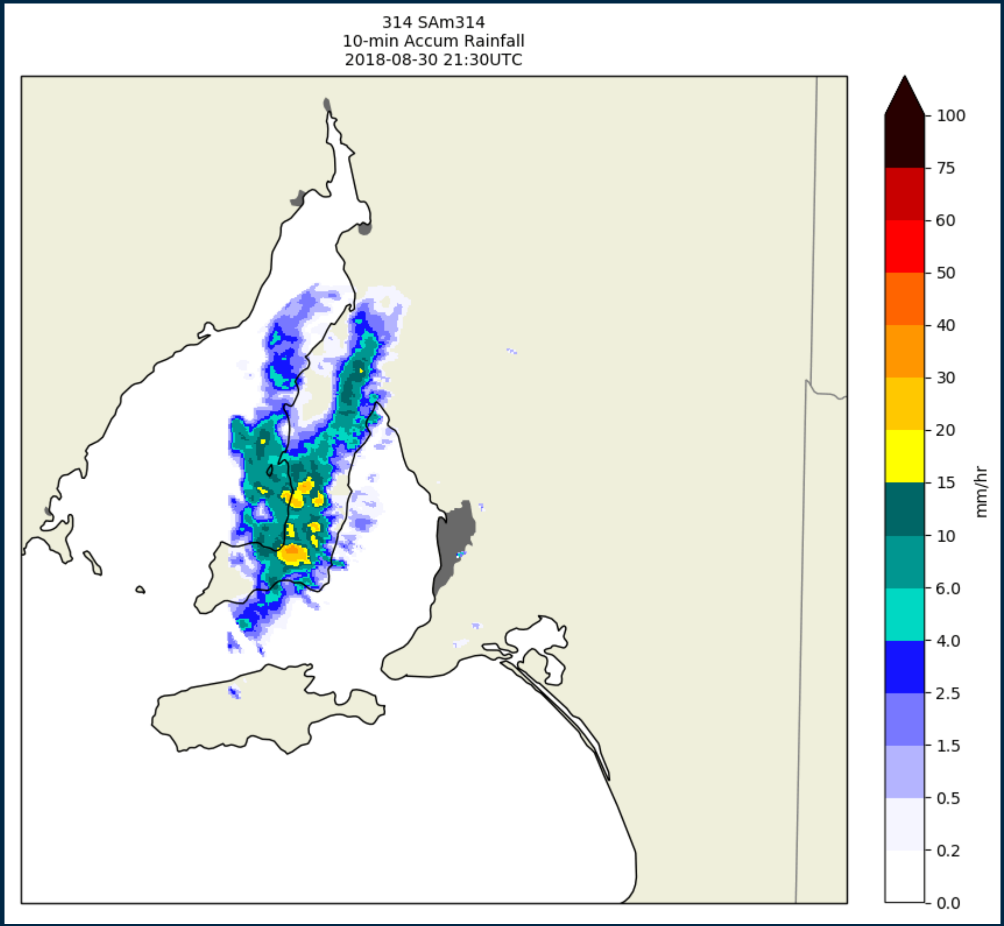
H_s is the scaling exponent, can be estimated using $\beta = 2 + 2H_s$
q is the scale ratio between cascade levels *k+1* and *k,* < 1
k is the level in the cascade with scale $l_k = q^k l_0$
cascade domain is l_0 x l_0

Conditional simulations

STEPS ensemble nowcasts

- Conditioned on radar data only
  - 30 member ensemble
  - Updated every 5 mins
  - 2 hour lead time
  - 5 min, 500 m resolution
  - 250 km domain
  - Adelaide, Melbourne, Sydney, Brisbane radars

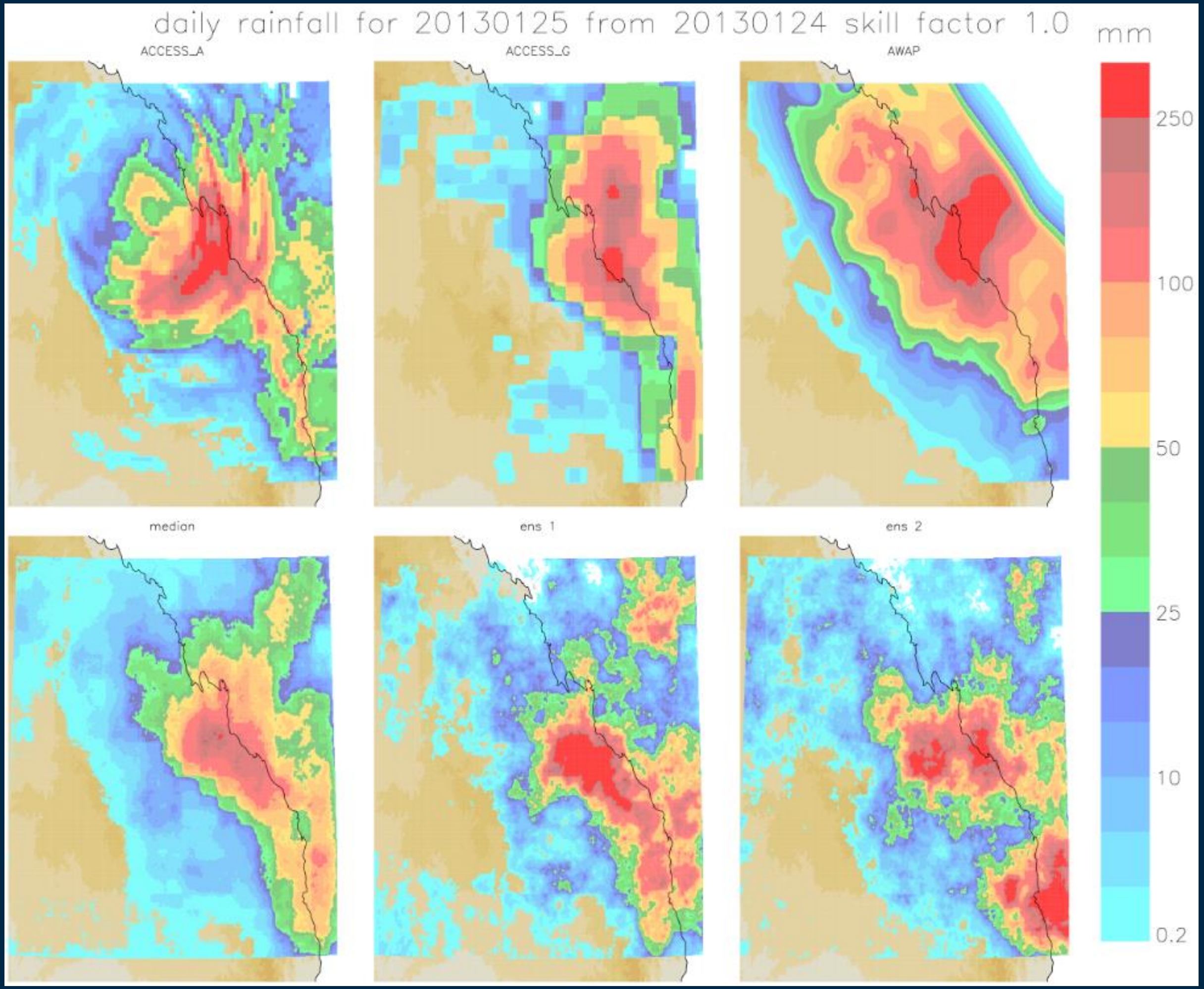
- Conditioned on radar and NWP forecasts
  - 30 member ensemble
  - Updated every 10 mins
  - 12 hour lead time
  - 10 min, 1 km resolution
  - 500 km domain
  - 7 domains



Downscale and blend NWP

Seamless rainfall

- Blend ACCESS-G & R
- 30 member ensemble
- Update 4x per day
- 5 day lead time
- 2 km, 1 hour resolution
- 1000 x 1000 km tiles
- 16 tiles over Australia



Daily rainfall accumulations – AWAP is the gauge analysis that is used as the "truth"

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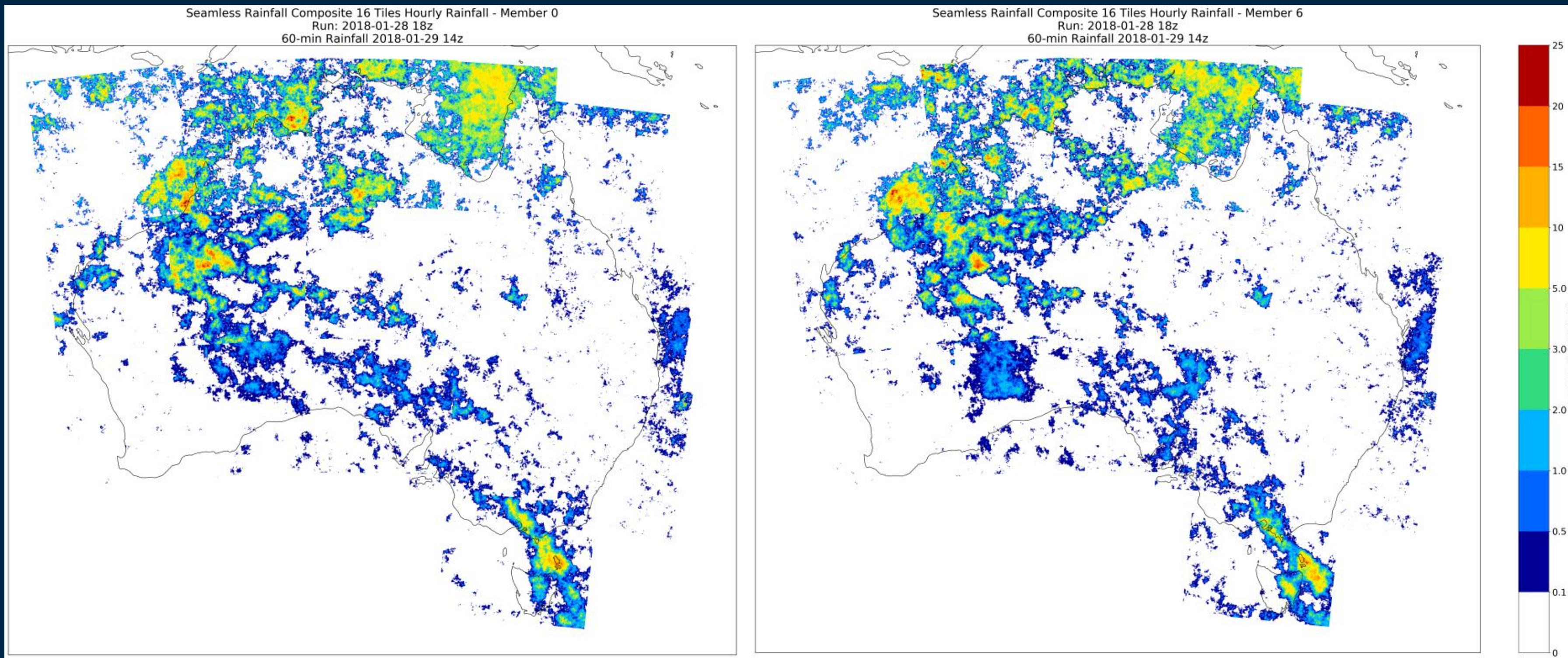
Seamless Rainfall products



Forecast hourly rainfall accumulation +20 hour
Composite ensemble members 0 & 6

Multi-sensor national hourly rainfall ensembles

- Work done by Renzullo (CSIRO Land and Water), and Velasco (Bureau of Meteorology)
- Objective is to generate an ensemble of rainfall fields that are conditioned on radar, NWP, satellite rainfall in real time
- To be used as input for flood forecasting and other applications
- Spread in the ensemble represents the uncertainty in the blended product

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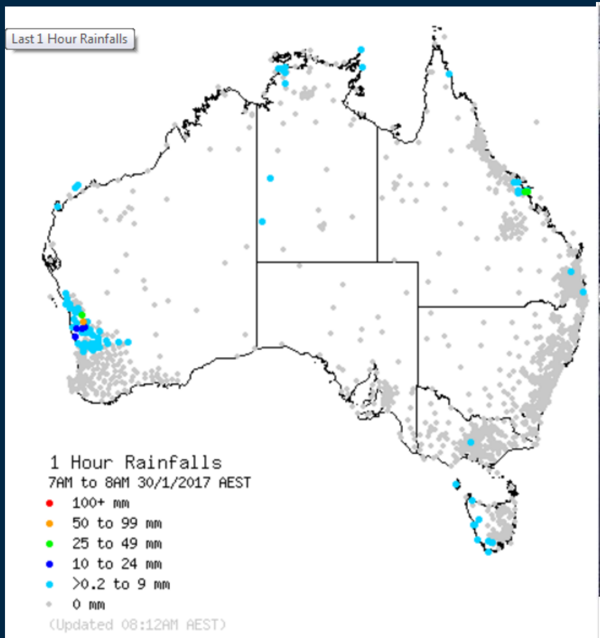
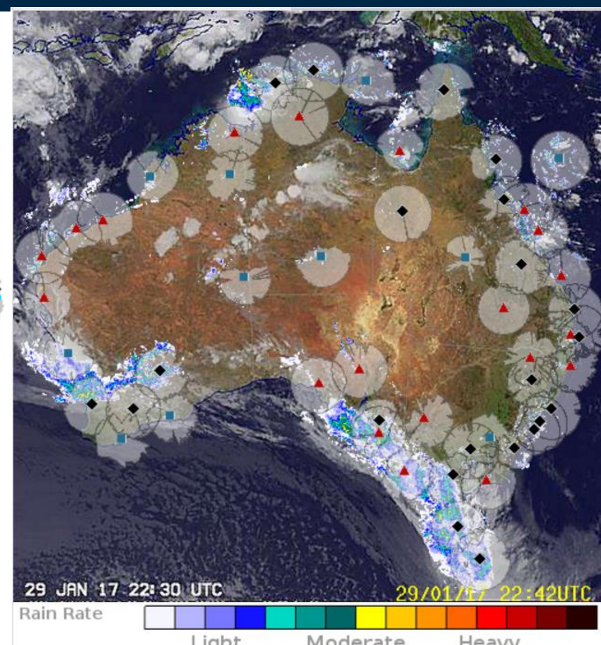
Blending multi-source gridded rainfall

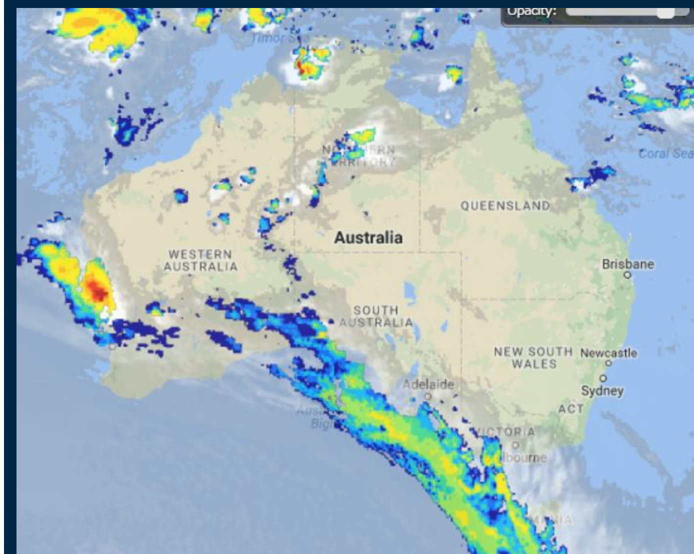
Rain Gauges

Weather Radar

Satellite - NWP







Real-time gauges

BoM Radars

IMERG (TRMM - GPM)
ACCESS – R
GSMap-NOW (Himawari-8)

Rainfields

Ensemble multi-sensor QPE

Renzullo and Velasco, 2017

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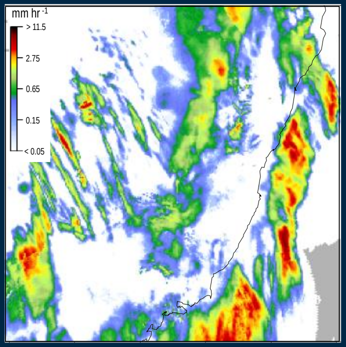
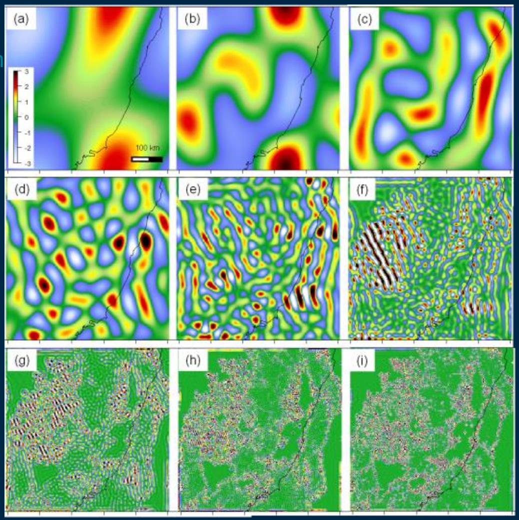
Blending multi-source gridded rainfall

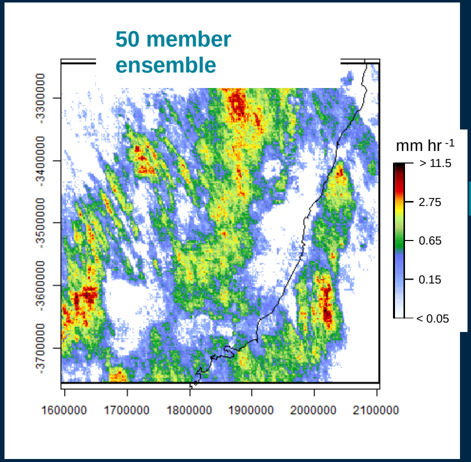
A Scale-dependent blending approach was explored

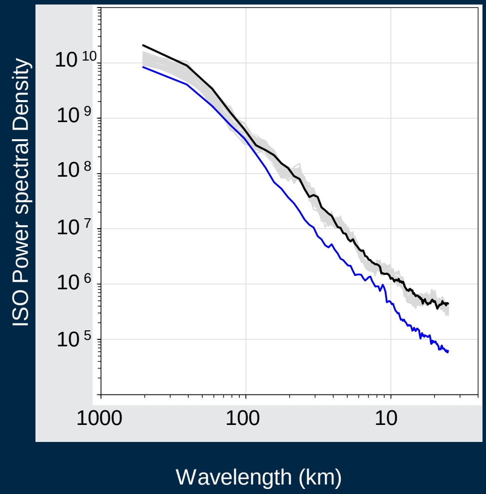
- Multiplicative Cascade modelling of rainfall (e.g. STEPS, Seed et al.)
- Fourier transform spectral decomposition
- Each rainfall data source is decomposed into spatial components
- Noise generated with the same structural properties as rainfall analysis
- Noise contribution to the blend increase with increasing cascade level to reflect the uncertainty is estimation at the finer spatial scales
- Components are weighted according to how well they represent rainfall at the give cascade level
- Weights of each source at each scale are calculated using an objective method based on triple collocation (Caires and Sterl, 2003; McColl et al., 2014), where three independent observations are used to infer the error variances in each respectively.



Spatial decomposition







Renzullo and Velasco, 2017

Semi-conditional continuous simulations

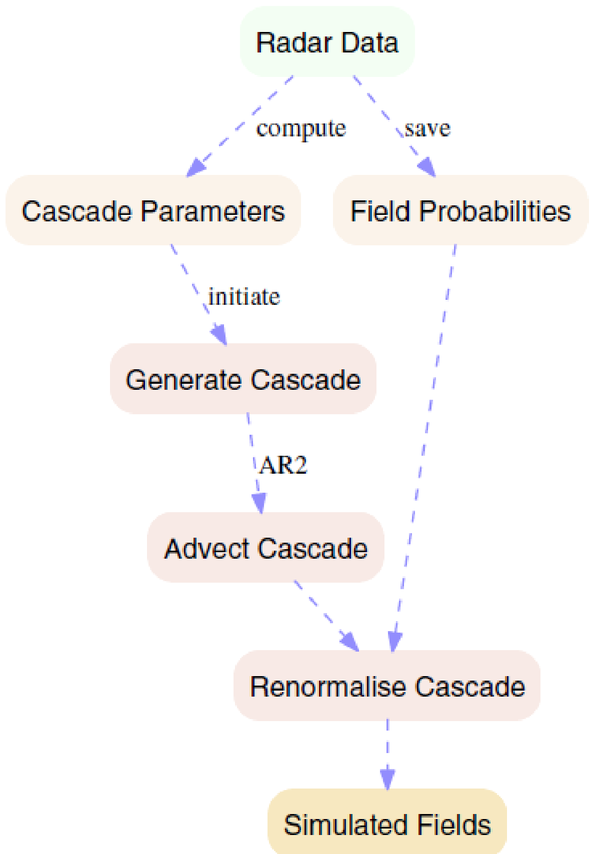


Figure 3. Flow chart of the Short-Term Ensemble Prediction System methodology used in this study. Radar scans at 6 min intervals are used to compute the cascade parameters, and the simulations are run 100 times for each set of parameters.

HiDRUS model developed by Raut at School of Earth Atmosphere Environment, Monash University

Model parameters are estimated from radar data

Ensembles used for urban infrastructure design and planning

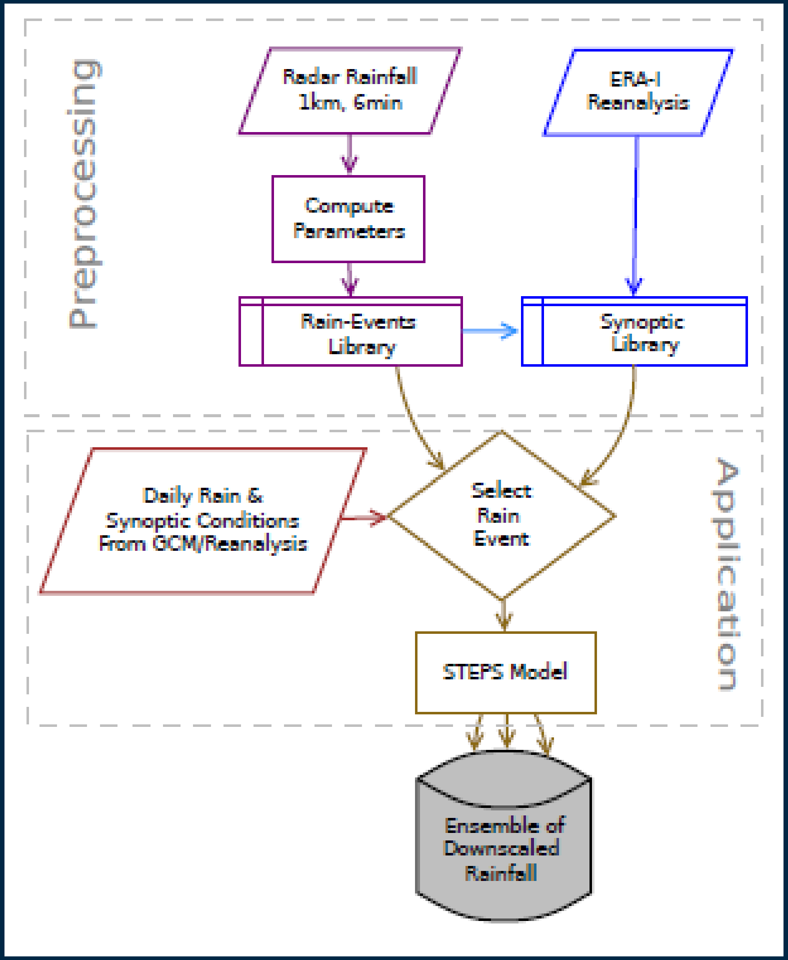
- 100 member ensemble
- 1 km, 6 min resolution
- 250 km domain
- 7-year period

Raut et al, 2018, JGR Atmospheres

Semi-conditional continuous simulations

Downscale ERA-1 reanalysis 1995 – 2004

- 100 member ensemble
- 1 km, 6 min resolution
- 250 km domain



Raut, 2018, pers com, Monash University

Unconditional event simulations

Model variables

Symbol	Variable
R	Mean areal rainfall over 256 km scale (mm/h)
σ_N	Standard deviation at the 1 km scale (mm/h)
Σ	Vector of the standard deviation for each level (8 levels) in the cascade
ρ_1	Vector of the lag 1 Lagrangian auto-correlations for each level in the cascade
ρ_2	Vector of the lag 2 Lagrangian auto-correlations for each level in the cascade
E	Advection east (km)
S	Advection south (km)
β_1	Slope of the spatial power spectrum for scales that are greater than 20 km
β_2	Slope of the spatial power spectrum for scales that are less than 20 km

Used to generate ensembles of design storms for Brisbane river catchment

- 10 member ensemble
- 1 km, 10 minute resolution
- 250 km domain
- 8 storms

Scaled each ensemble to match specific return periods over the catchment

Jordan et al, 2015, HWRS, Application of spatial and space-time patterns of design rainfall to design flood estimation

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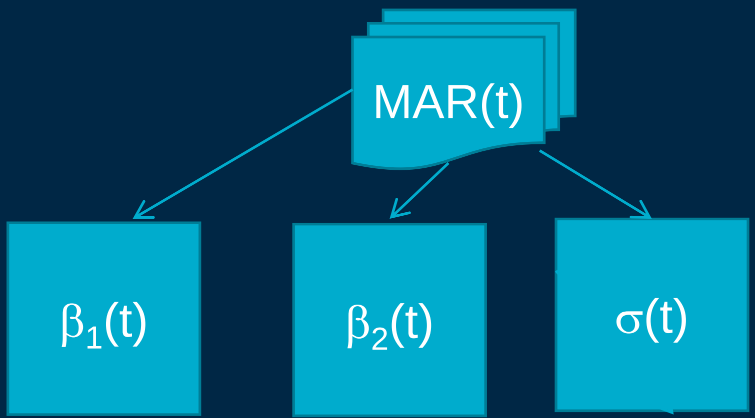
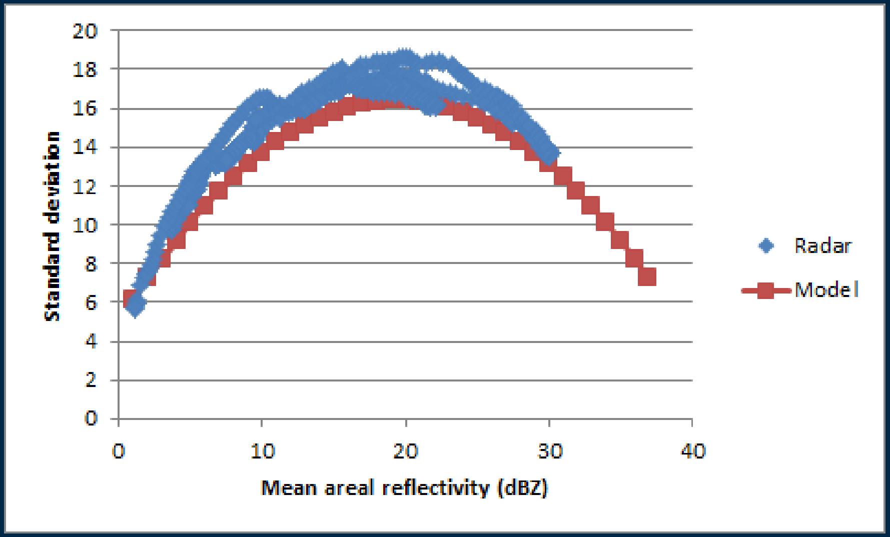
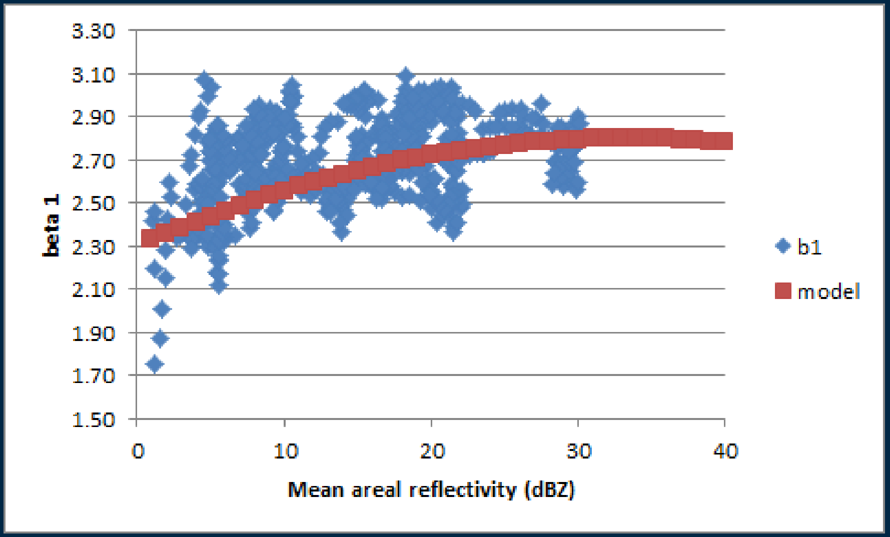
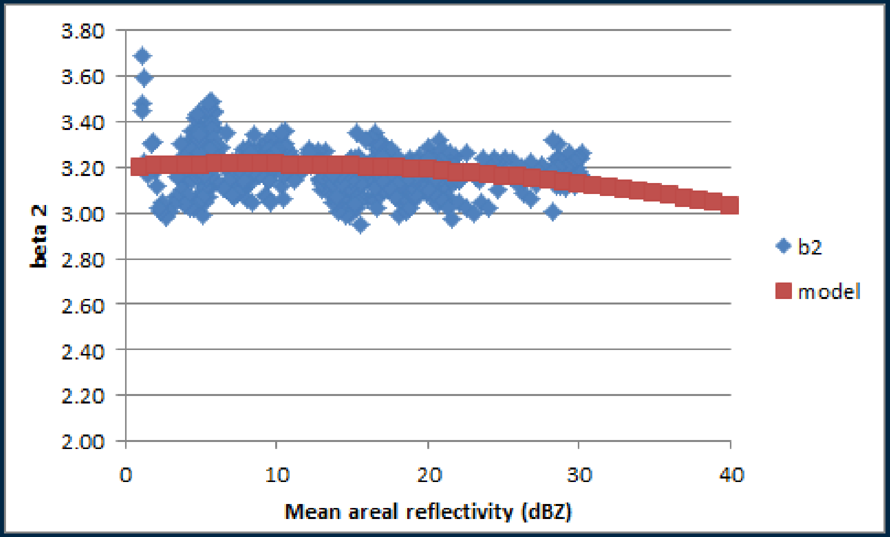
Time series of model variables









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Model parameters

Symbol	Parameter Description
μ_R, σ_R, A_R, H_R	Broken line for time series of mean areal rainfall in dBZ
μ_E, σ_E, A_E, H_E	Broken line for time series of advection east
μ_S, σ_S, A_S, H_S	Broken line for time series of advection south
a_v, b_v, c_v	Quadratic function to calculate the field standard deviation
a_1, b_1, c_1	Quadratic function to calculate β1
a_2, b_2, c_2	Quadratic function to calculate β2
a_t, b_t, c_t	Parameters to calculate ρ1, ρ2

Conclusions

- Rainfall has a very complex behaviour in space and time
- Bounded log-normal cascades can simulate a useful fraction of this behaviour
- STEPS has been used in a number of configurations to provide both conditional and unconditional simulations

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Thank you

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